

# Modeling Applied to Problem Solving

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**Abstract.** We describe a modeling approach to help students learn expert problem solving. Models are used to present and hierarchically organize the syllabus content and apply it to problem solving, but students do not develop and validate their own Models through guided discovery. Instead, students classify problems under the appropriate instructor-generated Model by selecting a system to consider and describing the interactions that are relevant to that system. We believe that this explicit System, Interactions and Model (S.I.M.) problem modeling strategy represents a key simplification and clarification of the widely disseminated modeling approach originated by Hestenes and collaborators. Our narrower focus allows modeling physics to be integrated into (as opposed to replacing) a typical introductory college mechanics course, while preserving the emphasis on understanding systems and interactions that is the essence of modeling. We have employed the approach in a three-week review course for MIT freshmen who received a D in the fall mechanics course with very encouraging results.

**Keywords:** physics education research, course reform, introductory physics, modeling instruction

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## INTRODUCTION

Standard mechanics teaching materials emphasize declarative and procedural knowledge, but not the strategic knowledge which is essential to the application of these facts and procedures in problem solving [1]. From this perspective the common student complaint “I understand the material, but I can’t start the problems” is a cry for help in gaining strategic knowledge. This paper presents a pedagogy for teaching Newtonian mechanics that is designed specifically to help students develop the strategic knowledge to apply course material to problem solving in an expert-like manner. We achieve this objective by helping students to organize the course material into a hierarchy, and to use this organization as a key component of the problem solving strategy.

Our approach is a simplification of the modeling physics approach [2, 3, 4] that is focused on helping students learn expert problem solving. We call our pedagogy Modeling Applied to Problem Solving (MAPS) to emphasize that its primary aim is enabling students to organize and activate their knowledge to solve problems. Restricting modeling to two goals – the organization of core course material and its application to problem solving – has several advantages. The narrowed focus implies a commensurately reduced set of new concepts and terminology, allowing MAPS to be more easily learned. The hierarchical organization helps students cope with the large amount of material covered in introductory mechanics. The explicit and universal problem modeling strategy places the emphasis on interactions as the key to physics models and their application in problem solving. Furthermore, MAPS has deliberately been structured in a manner that allows its incorporation into existing mechanics courses without significant alteration

of the syllabus or specialized training for the instructor.

We have implemented the ideas presented here in a one-month review course (hereafter the ReView course) involving 30 MIT students who received a grade of D in the fall 2008 semester introductory mechanics course.

## OVERVIEW OF MAPS PEDAGOGY

In MAPS, the scientifically established declarative and procedural knowledge of Newtonian Mechanics is organized into Models. These Models are process models [5], and are referred to as “basic models” in [3, 4]. We organize these Models into hierarchies under four physical categories (motion, momentum, energy, and angular momentum) to facilitate application to problem solving.

Strategic application of the Models is encouraged through the use of a problem modeling strategy. This strategy helps students represent a given physical situation in a manner that suggests the applicability of a particular basic Model. As in the problem-specific models of [3], it involves the description of the system under consideration and the interactions present. In our curriculum, we reserve the term “Model” for the instructor-generated basic models that are arranged in our hierarchy. The process of solution is called “modeling”.

**TABLE 1.** Contrasting the basic Models of the hierarchy with the problem-specific modeling process.

<b>Models</b>	<b>modeling</b>
Theory.	Application.
General principle.	Problem-specific.
Organize syllabus.	Apply content.
Exact, ideal.	Approximating, idealizing.
Existing knowledge.	Student-constructed.

## SPECIFICATION OF MODELS

MAPS pedagogy uses general physical Models to present the basic physical principles of Newtonian mechanics. These models function as cognitive “chunks” [6] that link the many disparate facts and formulae related to a particular concept. They are specified according to the template:

**Compatible Systems:** Systems which can be described by the Model. Some Models can only describe a single point particle, others can describe the behavior of a collection of rigid bodies.

**Relevant Interactions:** The class of interaction which serves as the agent of change in the Model. Models require the separation of interactions into relevant and irrelevant types based upon classifications like conservative vs. non-conservative, internal vs. external, and torque-producing vs. non-torque-producing.

**Law of Change:** The heart of each Model is the equation or equations which describe the evolution of the state of the system if the Model holds [3].

A sample Model specification for Momentum and External Force is shown below.

### MOMENTUM AND EXTERNAL FORCE

**Compatible Systems:** The system may contain any number of point particles. Rigid bodies may be treated as point particles with positions specified by their centers of mass.

**Relevant Interactions:** Only external forces need to be considered. Internal forces do not change the system’s momentum.

**Laws of Change:**

**Integral**

$$\sum_{\text{sys}} \vec{p}_f = \sum_{\text{sys}} \vec{p}_i + \int \sum_{\text{ext}} \vec{F} dt$$

**Differential**

$$\sum_{\text{sys}} \frac{d\vec{p}}{dt} = \sum_{\text{ext}} \vec{F}$$

## HIERARCHY OF MODELS

To help the student build an overview of the entire domain, we categorize each Model under a basic physical category. The typical calculus-track course is concerned with four such categories:

- Motion, Acceleration and Net Force
- Momentum and External Force
- Mechanical Energy and Non-Conservative Work
- Angular Momentum and External Torque

These categories explicitly indicate the relevant type of interaction for all Models that fall into the category, emphasizing to the students that the type of interaction is their guide to choosing a Model when solving a problem, and reminding them that only certain types of interactions function as agents of change for a given Model. This association of a class of relevant interactions with each physics principle is the most important vehicle for the recognition and application of the principles when solving problems.

Within these four categories, a hierarchy of detailed Models is constructed that connect the broad physical principles with more specialized recurring patterns of nature. One possible detailed hierarchy for the Motion, Acceleration and Net Force category is shown below. Models that are specializations of another Model are indented beneath the more general Model.

Motion, Acceleration and Net Force (3-D)

General 2-D Motion

Rotational Motion

Uniform Circular Motion

General 1-D Motion

1-D Motion with Constant Acceleration

Simple Harmonic Motion

## PROBLEM MODELING STRATEGY

In the MAPS pedagogy, all problems are approached using the S.I.M. problem modeling strategy: System, Interactions, and Model(s). The strategy is central to the main objective of MAPS: helping students become better problem solvers. Its goals can be summarized as *strategic*, *universal*, and *simple*:

- Give students an expert-like strategic approach for finding the relevant physical model and associated equations.
- Keep the steps as universal as possible for better transfer to other domains.
- Make it easy to learn, implement, and incorporate into the pedagogy of an existing course.

Experts possess the insight to sort problems into categories based upon their interactions. They recognize that if no external forces are applied to a system then momentum will be conserved, if non-conservative forces are absent or do no work then mechanical energy is conserved, and so on. Novice students also look for conserved quantities, but do so by using superficial features and equation hunting rather than attempting to qualitatively understand the nature of the interactions [6, 7, 8]. For this reason, when novices are presented with a problem that they do not recognize based on pattern matching to previous problems or equations, they are helpless to proceed.

### The S.I.M. Problem Modeling Strategy

**Description of the System:** Students should list all the objects that they consider to be part of the system and classify them as point particles, rigid bodies, or massless objects.

**Description of Interactions:** Every problem requires a careful description of the interactions. Students should classify every relevant force as internal/external and conservative/non-conservative, since this is the key to choosing the model which will most efficiently solve the problem.

**Choice of Model:** A selection from the hierarchy. The consistency of the three parts of the strategy shows whether the student has made a reasoned choice without requiring a detailed explanation.

The S.I.M. strategy promotes an expert approach to mechanics problems because it fosters a strong understanding of the interplay between the description of the system, the nature of the interactions, and the choice of the appropriate basic physical Model. The most important interdependencies are:

- The description of the system dictates which forces will be internal versus external versus excluded.
- The type of forces acting (conservative, internal, torque-producing, etc.) influences which physical variables are good descriptors, and hence which Model(s) will apply.
- The choice of Model dictates which interactions are relevant to, and what system descriptions are compatible with, the Law of Change governing the system's evolution.

With these interdependencies in mind, it is impossible to view the specification of the system, the description of the interactions and the choice of the Model as a linear process (e.g. first system, then interactions then Model). These interdependencies are at the heart of strategic problem solving. By making an insightful choice when specifying the system or choosing the model, the relevant interactions can sometimes be greatly simplified.

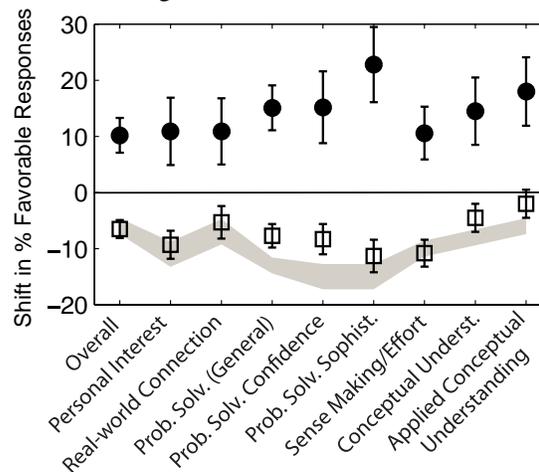
The S.I.M. strategy is simplified relative to problem solving in a typical modeling physics curriculum without sacrificing the strategic core of the approach. Because it is universally applicable to any mechanics problem, it has the advantage of setting up a clear procedure for the students to follow when they are initially stuck. The S.I.M. strategy is succinct enough to be habitually used by an instructor on all example problems worked during class, and to constitute a minimal additional workload in written solutions for the students.

## OUTCOMES

### Improved Student Attitudes

In January 2009 we taught a three-week ReView for 30 students who received a D in the fall 2008 MIT mechanics course using the MAPS pedagogy. Students taking the ReView and achieving a sufficiently improved grade on an equivalent final exam given at its conclusion had their grade raised to C, giving credit for the course.

In contrast with most introductory physics courses (including the MIT course) which leave students with *more novice* attitudes about physics and problem solving than they had when they entered [9, 10], our ReView generated favorable shifts in every category of the Colorado Learning Attitudes about Science Survey (C-LASS) [10] (Fig. 1). These shifts were significant at the  $2\sigma$  level in all five categories related to problem solving and conceptual understanding.



**FIGURE 1.** C-LASS shifts by category for the ReView (black circles), MIT 8.01 (squares) [105 voluntary respondents] and a typical university course (gray band) [10].

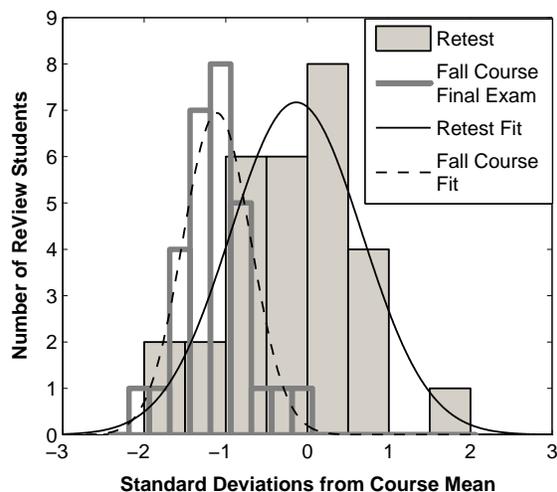
The student responses to the course evaluations strongly suggest that this attitude shift was largely brought about by the problem solving approach employed. Student free-responses to the questions “What one or two things did you like about this course?” and “Do you feel you are ready to move on to [the second semester course]?” yielded comments including:

“The S.I.M. rubric was extremely helpful. It should be taught in [mechanics] the first time around.”

“I liked...looking at the basic solving steps that are normally assumed...”

“The systems, interactions, models stuff makes you think before jumping in.”

Overall, 15 of the 33 respondents offered explicit endorsements of MAPS similar to those reported above.



**FIGURE 2.** Original and retest final exam histograms for the ReView. The lines represent Gaussian fits to the data.

### Improved Student Performance

One measure of performance shift for our ReView course is a comparison of the students' exam scores from the fall term with their performance on the final exam retest administered upon completion of the ReView. To allow for the most valid comparison possible, we used a final exam given to a previous freshman class as our retest. This gave us a set of 571 scores from which to generate a curve for the test. Since a comparison of the absolute score on the retest with the absolute score on the 2008 test is not valid, we made the assumption that the abilities of the 2008 freshmen were identical in distribution to the abilities of the freshmen from the year the retest was used as the final exam, allowing us to make a valid comparison of z-scores. The results, shown in Fig. 2, indicate a substantially improved performance for the ReView students as a group. The mean z-score of the 29 students on the fall 2008 exam was  $-1.11 \pm 0.08$  while their mean z-score on the retest was  $-0.13 \pm 0.15$  (within error of the overall average for the full freshman class).

Another measure is student performance on the Mechanics Baseline Test [11]. The three-week ReView achieved a normalized gain of  $0.20 \pm 0.07$ , more than half the normalized gain achieved by the same students during the one-semester fall course ( $0.38 \pm 0.06$ ). The gain achieved was sufficient to bring the mean gain of the ReView students for the fall course and ReView combined ( $0.52 \pm 0.04$ ) into correspondence with the overall mean gain for the fall course ( $0.48 \pm 0.01$ ).

### CONCLUSIONS

The MAPS pedagogy outlined in this paper proved successful in significantly improving student attitudes as

measured by the C-LASS and performance as measured by an MIT final exam and the Mechanics Baseline Test among a group of 30 D students taking a three-week ReView course offered after the regular mechanics course. These results are extremely encouraging, but it must be noted that the test group was strongly motivated to alter their problem solving approach and already knew much of the course content, permitting them to concentrate on organizing the material rather than learning it. In fall 2009 we will employ the approach in one section of the regular freshman course at MIT to investigate whether similar results can be achieved in a group learning the MAPS approach concurrently with the course content.

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