## Conversion Formula:

Consider a liquidity pool with two reserve tokens S and R of equal weights, where:

- $s =$ balance of token S
- $r =$ balance of token R

Converting an amount of S tokens into R tokens is based on $y = r \cdot \left(1 - \frac{s}{s+x}\right)$, where:

- $x =$ input amount of token S
- $y =$ output amount of token R

The conversion formula for any type of weighted-pool is $y = r \cdot \left(1 - \left(\frac{s}{s+x}\right)^{w_1/w_2}\right)$, where:

- $w_1 =$ weight of token S
- $w_2 =$ weight of token R

As you can see, when $w_1 = w_2$, the latter formula reduces to the former formula.

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## Calculating New Weights:

Let F and G denote the primary reserve token and the secondary reserve token respectively.

Let 'on-chain price' denote the conversion rate between F and G inside the pool (i.e., as determined by the pool).

Let 'off-chain price' denote the conversion rate between F and G outside the pool (i.e., as determined by the market).

Let the following denote:

- $t =$ F staked balance of the pool
- $s =$ F reserve balance of the pool
- $r =$ G reserve balance of the pool
- $q =$ G/F off-chain price numerator
- $p =$ G/F off-chain price denominator

Where 1 unit of F is equal to $q/p$ units of G (or $p$ units of F are equal to $q$ units of G).

First, note that the market's arbitrage incentive is always to convert units of F to units of G or vice-versa, such that the on-chain price of F/G will become equal to the off-chain price of F/G.

Consider the case of $t > s$. Our goal is to set the weights of the pool, such that the arbitrage incentive of equalizing the on-chain price and the off-chain price will subsequently increase $s$ to become equal to $t$. In other words, we want the arbitrager to transfer $t - s$ units of F to the pool, in exchange for units of G.

Suppose that we've set the weights:

- $w_1 =$ F reserve weight
- $w_2 =$ G reserve weight

Then:

- A user converting $t - s$ units of F will get $r \cdot \left(1 - \left(\frac{s}{t}\right)^{w_1/w_2}\right)$ units of G
- F reserve balance after the arbitrage conversion will be $t$ of course
- G reserve balance after the arbitrage conversion will be $r - r \cdot \left(1 - \left(\frac{s}{t}\right)^{w_1/w_2}\right)$
- F/G on-chain price after the arbitrage conversion will be $\frac{t \cdot w_2/w_1}{r - r \cdot \left(1 - \left(\frac{s}{t}\right)^{w_1/w_2}\right)}$
- F/G off-chain price is of course $\frac{p}{q}$ (or $\frac{q}{p}$ if the inverse rates are provided)

When either $t$ or $p/q$ change, we want to recalculate $w_1$ and $w_2$ such that the arbitrage incentive of making the on-chain price equal to the off-chain price will be equivalent to converting $t - s$ units of F to units of G, thus increasing F reserve balance ($s$) to be equal to F staked balance ($t$).

In other words, we want to recalculate $w_1$ and $w_2$ such that $\frac{t \cdot w_2/w_1}{r - r \cdot \left(1 - \left(\frac{s}{t}\right)^{w_1/w_2}\right)} = \frac{p}{q}$.

Let $x$ denote $w_1/w_2$, then:

$$\frac{t/x}{r - r \cdot \left(1 - \left(\frac{s}{t}\right)^{x}\right)} = \frac{p}{q} \rightarrow$$

$$\frac{t/x}{r \cdot \left(\frac{s}{t}\right)^{x}} = \frac{p}{q} \rightarrow$$

$$x \cdot \left(\frac{s}{t}\right)^{x} = \frac{tq}{rp} \rightarrow$$

$$x = \frac{W\left(\log\left(\frac{s}{t}\right) \cdot \frac{tq}{rp}\right)}{\log\left(\frac{s}{t}\right)}$$, where W is the Lambert W Function.

After computing $x$, we can represent it as a quotient of integers, i.e., $x = a/b$.

Then, since $x = w_1/w_2$ and $w_2 = 1 - w_1$, we can calculate:

- $w_1 = \frac{x}{x+1} = \frac{a/b}{1+a/b} = \frac{a}{a+b}$
- $w_2 = \frac{1}{x+1} = \frac{1}{1+a/b} = \frac{b}{a+b}$